Every Student Counts

Professional Development Guide
High School Level

Year 1 - Day 2

Iowa Department of Education

Every Student Counts – High School Facilitator Plan
DAY 2 of Year 1

Content Goal: NCTM Algebra Standard – Represent and analyze mathematical situations and structures using algebraic symbols
Process Goal: NCTM Reasoning and Proof Standard
Overall Teaching Goal: Teaching and learning mathematics through problem solving

TM = Teaching Master
● = materials or equipment

Activity	Description for Facilitator	Time (Min)	Teacher Masters (TM) & Materials
1. Introduction and Overview	Overview of Day 2	10	**TM-1** Year 1 Outline Chart **TM-2** Day 2 Overview Chart **TM-3** Day 2 Agenda
2. Meaningful Distributed Practice	Practice problems from Day 1 – individuals solve two problems, brief discussion	10	**TM-4** Distributed Practice handout
3. Readings Discussion	Readings summary and discussion – round robin group work, points posted on walls	30	**TM-5** Day 2 Reading Assn. • Chart paper • Pens • Tape
4. Algebra Problem-Based Instructional Task 1	Solving linear equations – group work followed by discussion	50	**TM-6** Solving Linear Equations handout • Graphing calculators • Graph paper
5. Algebra Reasoning Task 1	Reasoning about $(a+b)^2$ – groups analyze student work and provide reasoning behind geometric "proofs without words"	45	**TM-7** Analyzing Student Work - $(a+b)^2=a^2+b^2$ overhead • Internet connection **TM-8** Squaring a Binomial handout **TM-9** Proofs without Words handout
6. Analyzing Student Work	Participants bring in sample student work, groups analyze and discuss	35	• Student work from participants **TM-10** Analyzing Student Work overhead.
7. Algebra Problem-Based Instructional Task 2	Solving quadratic equations – group work followed by discussion	55	**TM-11** Solving Quadratic Equations handout **TM-12** Quadratic Formula handout • Graphing calculators • Graph paper
8. Algebra Reasoning Task 2	Completing the square – presentation, group work, discussion, "proof without words"	30	**TM-13** Completing the Square overhead **TM-14** Completing the Square handout • Internet connection
9. Reflecting on the Day	Look back at the day's activities through the lens of the Readings	20	**TM-15** Chart paper lists of main points from Readings Discussion activity above
Assignments	Distribute and discuss the Reading and Practice assignments	10	**TM-16** Reading Assignment handout **TM-17** Practice Assignment handout

Activity 1. Introduction and Overview of Day

Time: 10 minutes

Rationale
This activity presents an overview of the day's topics and activities.

Connections
This activity connects places the day's topics and activities within the context of the year.

Conducting the Activity
Begin the day by briefly presenting and discussing the following three handouts:

1. Year 1 Outline Chart
   - Remind participants of the big picture for the year.
   - Point out where we've been and where we're going.

2. Day 2 Overview Chart
   - Remind participants of the three main themes of Every Student Counts: teaching for understanding, problem-based instructional tasks, and meaningful distributed practice.
   - Point out how those three themes will be applied to the two focus points for Day 2: algebraic symbols and reasoning & proof.
   - Point to the list of main Day 2 activities at the bottom of the page.

3. Day 2 Agenda
   - Briefly go through agenda.

Materials
Handouts (pages 4 – 6)

TM -1 (Handout)

Year 1 Outline

Content Focus: Algebra
Teaching Focus: Teaching Mathematics through Problem Solving

	Day 1	Day 2	Day 3	Day 4	Day 5
Algebra Theme (NCTM Content Standards)	Functions	Algebraic Symbols	Mathematical Models	Linearity	Rate of Change
Teaching Theme (NCTM Process Standards)	Commu-ication	Reasoning and Proof	Problem Solving	Connections	Representation
Mathematical Activities	Investigating functions, especially from a recursive point of view	Making sense of algebraic symbols and mathematical proof	Using mathematical modeling to solve problems in real-world contexts	Investigating linearity in various mathematical settings	Investigating rate of change as a fundamental theme in mathematics
Technology (Graphing calculators each day)	Applets Spreadsheets	Applets Computer Algebra Systems	Spreadsheets	Applets	Calculator-Based Data Collection Devices

Reading assignments each day will include: (1) the Process Standard for the next day, (2) the particular Algebra Standard Goal for the current day, and (3) related research and teaching readings.

TM 2 (Handout)

Day 2 Overview Chart

Every Student Counts means...

Teach for Understanding & Focus on Meaning *using . . .*

Problem-Based Instructional Tasks & Teaching through Problem Solving

Meaningful Distributed Practice of Concepts, Skills, and Problem Solving

Today's Goals...

Content Goal: *Algebraic Symbols*

Process Goal: *Reasoning and Proof*

Today's Objectives...

- *Solve linear equations*
- *Solve quadratic equations*
- *Manipulate, represent, and interpret quadratic expressions*
- *Reasoning and proof with quadratic expressions*

TM 3 (Handout)

Day 2 Agenda

Algebra Theme: Algebraic Symbols – Represent and analyze mathematical situations and structures using algebraic symbols

Teaching Theme: Reasoning and Proof

1. Introduction and Overview of the Day
2. Meaningful Distributed Practice from Day 1
3. Readings Discussion – round robin
4. Algebra Problem-Based Instructional Task 1 – Solving Linear Equations
5. Algebra Reasoning Task 1 – Reasoning about $(A + B)^2$
6. Analyzing Student Work
7. Algebra Problem-Based Instructional Task 2 – Solving Quadratic Equations
8. Algebra Reasoning Task 2 – Completing the Square
9. Reflecting on the Day – from the perspective of the readings
10. Assignments and Evaluation

Assignments:

a. Reading (see attached)

b. Practice Assignment:

Observe at least one algebra lesson in your collaborating teacher's classroom.

- Take notes so that you can describe the lesson during the Day 3 workshop.
- If the lesson illustrates a problem-based instructional task, as in the Marcus and Fey article (see Part a above), explain how so.
- In any case, pose a problem, as in the Goldenberg and Walter article (see Part a above), that would enrich the lesson.
- In particular, examine Table 6.1 on pp. 75-77, choose at least one of the "Useful Questions" in the left column, and generate a problem derived from the context of the lesson you observed, as in the right column of the table.
- Be prepared to describe and discuss the lesson and your proposed problem during the Day 3 workshop.

Activity 2. Meaningful Distributed Practice

Time: 10 minutes

Overview and Rationale

This activity provides examples and discussion of meaningful distributed practice. Meaningful distributed practice is one of the three main themes of the Every Student Counts program, along with teaching for understanding and problem-based instructional tasks.

Connections

This is the first activity of the day, modeling how teachers would use meaningful distributed practice at the beginning of a classroom day.

Conducting the Activity

- **Grouping**
  Participants work individually on two meaningful distributed practice problems from the Day 1 workshop.

- **Introduction**
  Introduce the activity as modeling meaningful distributed practice in the classroom.

- **Problem Solving (5 minutes)**
  Individuals solve the two problems. When finished, they can discuss their solutions quietly with their neighbors until everyone is finished.

- **Discussion (5 minutes)**
  The main focus of this activity is to model classroom use of meaningful distributed practice. Thus, the discussion of the solutions should be very brief. Then, getting back to the role of teachers, have a brief discussion of why these problems are examples of meaningful distributed practice of basic ideas from Day 1.

Materials

- Distributed Practice Handout (or simply write the problems on the board or overhead)

TM 4 (Handout)

Distributed Practice

Take a few minutes to solve the two problems below.

1. Examine the table below. Write an equation using NEXT and NOW that describes the pattern in y-values.

x	***y***
0	6
1	10
2	14
3	18
4	22

2. Consider this equation: NEXT = 2 NOW. Construct a table that has the pattern expressed by this equation.

Activity 3. Discussion of Assigned Readings

Time: 30 Minutes

Overview and Rationale

This small group activity gives participants the opportunity to identify and briefly discuss the main points in the assigned readings. Doing this early in the day sets the stage for connecting the day's activities to the readings whenever feasible.

We will refer to these points throughout the day. At the end of the day, we will reflect back on the day's activities through the lens of the readings. (See "Reflections on the Day through the Readings" activity.)

Connections to Other Activities and the Whole Day

This activity comes early in the day so that we can refer to the main points of the assigned readings throughout the day. This activity is a companion to an one at the end of the day in which participants return to the main points identified in this activity so that each group can reflect on the day in terms of each reading.

Conducting the Activity

- **Grouping**
  Five (or a multiple of five) groups are formed so that at least one person with secondary mathematics teaching preparation is in each group. (If necessary, the two "Reasoning and Proof" readings can be combined into one for discussion purposes, thus requiring four, or a multiple of four, groups.)
- **Discussion Method**
  Each group is initially assigned one of the five readings. They generate main points, important questions, and/or key examples from that reading and write these on a sheet of chart paper. After 5 minutes, the sheets are passed on to an adjacent group. Each group reads the points that the previous group listed for the new reading and then adds more points, questions, or examples. This process continues until each group has considered each reading. All charts are taped to the wall for viewing during the rest of the day and for reference during the afternoon summary activity.
- **Time**
  Allow 5 minutes to transition to and from the activity, and 5 minutes for each group to discuss each reading. Total time: 30 minutes.

Materials

- Chart paper, pens, and tape for each group

Reading Assignment for Day 2

1. The algebraic focus for Day 1 was Functions.

   - Read the sub-section in the Algebra Standard for Grades 9 – 12 entitled "Understand patterns, relations, and functions" in *Principles and Standards for School Mathematics*, pages 297-300.
   - This reading describes the main algebraic ideas that were the focus of Day 1.
   - Reflection Question: How does the view of algebra and functions in this reading relate to the activities that you completed on Day 1?

2. The benefits and barriers to teaching mathematics through problem solving, the overall teaching focus for the ESC program, are discussed from different perspectives in the following readings.

   - Read Chapter 13, "Phasing Problem-Based Teaching into a Traditional Educational Environment" by Copes & Shager in *Teaching Mathematics through Problem Solving Grades 6 – 12*, pages 195 – 205.
   - As the title indicates, this reading addresses the challenges and benefits of a gradual introduction of teaching through problem solving into a traditional classroom setting.
   - Reflection Question: According to these authors, how can teaching through problem solving be phased into a traditional mathematics classroom?

   - Read Chapter 2, "What Research Says About the NCTM Standards" by Hiebert in *A Research Companion to Principles and Standards for School Mathematics*, pages 5 – 23. Copy of the chapter is in your ESC Notebook.
   - This chapter is a readable and complete summary of the research base for *Principles and Standards for School Mathematics*.
   - Reflection Question: According to research, how does what students often learn in traditional mathematics classes compare to what they learn in alternative (NCTM Standards-based) programs?

3. Reasoning and Proof will be the teaching theme for Day 2.

   - Read "Reasoning and Proof" for Grades Pre-K – 12 (pp. 56–59) and for Grades 9 –12 (pp. 342-346) in *Principles and Standards for School Mathematics*.
   - Be prepared to analyze the analyze the Day 2 activities in terms of the four goals of the Reasoning and Proof Standard.
   - Reflection Question: According to these readings, what is the teacher's role in developing reasoning and proof in mathematics classes?

Activity 4. Algebra Problem-Based Instructional Task 1: Solving Linear Equations

Time: 50 minutes

Overview and Rationale

During this day's focus on algebraic symbols, we will investigate two classic problems in high school algebra–solving linear equations and solving quadratic equations. We will see how these formal tasks using algebraic symbols can and should be carried out meaningfully and with a problem-based approach. This solving linear equations task emphasizes multiple solution strategies. First, participants solve a simple linear equation in as many ways as they can, then they systematically discuss and analyze 5 specific strategies. There are two versions of this task: (1) the workshop only version, which can be used in workshops with teachers, and (2) the workshop and student version, which can be used in workshops with teachers and with students in classrooms.

Connection to Other Activities and the Whole Day

This activity begins the day's algebra activities. It illustrates well the "algebraic symbols" focus for the day, as well as reasoning. It will be complemented by the solving quadratic equations task later in the day.

Conducting the Activity

- **Grouping**
  Groups are formed so that at least one person with secondary mathematics teaching preparation is in each group.
- **Introduction**
  Briefly discuss the points above in the "Overview" and "Connection" sections. Decide which version you will use, either (1) the workshop only version, which can be used in workshops with teachers, or (2) the workshop and student version, which can be used in workshops with teachers and with students in classrooms. The description below is for version (1). It should be slightly modified as per the format of version (2) if that version is used.
- **First Part**
  First, state the linear equation: 3x+4=10, and ask the groups to solve this equation in as many ways as they can. Then let participants briefly report on the methods they used. Keep this brief.
- **Second Part**
  Next, hand out the "Solving Linear Equations" activity sheet. Point out that they have already done Part a. Now they should work in groups to complete Part b. After most groups have finished, discuss the strategies by having one group describe their work with each strategy.

Continues on next page.

Activity 4 continues:

- **Closure**
  Be sure to highlight the following points: (a) multiple solution strategies and multiple representations, (b) why are these multiple approaches important - access, builds connections, develops deeper more robust understanding, develops skill with strategies that may work better than the standard symbolic manipulation strategy on some problems, (c) even classic symbolic problems have meaning–in terms of the symbols and in terms of other representations, (d) note that the symbol manipulation approach to solving this simple linear equation generalizes to other equations that are in linear form, e.g., $3\sin x+4=10$, and (e) the symbol manipulation strategy has severe limitations when the equations get more complex.
- **Time**
  Allow 5 minutes to introduce the activity, 10 minutes work and 5 minutes discussion for the first part (Part a), 20 minutes work and 10 minutes discussion for the second part (Part b). Total: 50 minutes.

Materials

- "Solving Linear Equations" Handout, "Solving Linear Equations (V2)" Handout, graphing calculators, graph paper

TM 6 (Handout)

Solving Linear Equations

Multiple Solution Strategies

Solve: 3x + 4 = 10

(a) Devise and carry out as many solution strategies as you can.

(b) For each of the solution strategies below:

- Put a checkmark next to the strategy if it is one that you used.
- If you did not already use that solution strategy, carry it out now.
- Discuss some advantages and disadvantages of each strategy.
- Discuss some teaching and learning issues related to each strategy. For example: Do you think a student might use this strategy? What would you do as the teacher if you saw a student using this strategy? Would you explicitly teach this strategy?

(c) Did you use a strategy that is not listed below? If so, add it to the list and discuss it as above.

Solution Strategy 1: By Inspection (or Guess and Check)

Is there a number such that when you multiply it by 3 and then add 4 you get 10?
That is, fill in the box: 3 * θ + 4 = 10.

Advantages:

Disadvantages:

Teaching and Learning Comments:

Solution Strategy 2: Standard Symbolic Manipulation

Subtract 4 from both sides, then divide both sides by 3.

Advantages:

Disadvantages:

Teaching and Learning Comments:

Continues on next page.

Continues from prior page.

Solution Strategy 3: Non-Standard Symbolic Manipulation

Divide both sides by 3. Then continue manipulating until you get x = θ.

Advantages:

Disadvantages:

Teaching and Learning Comments:

Solution Strategy 4: Table

Consider y = 3x + 4. Enter this function into your calculator. Set up a table with proper step size and range so that you can locate the solution to the equation 3x + 4 = 10.

Advantages:

Disadvantages:

Teaching and Learning Comments:

Solution Strategy 5: Graph

Consider two functions: y = 3x + 4 and y = 10. Enter these two functions into your calculator. Graph in the proper window so that you can locate the solution to the equation 3x + 4 = 10.

Advantages:

Disadvantages:

Teaching and Learning Comments:

Solving Linear Equations (V2)
Multiple Solution Strategies

LAUNCH

Consider the equation: $2x + 1 = 7$.

a. What does the equation mean?

b. What does it mean to "solve" the equation?

c. What is the solution? How do you know?

EXPLORE

Consider the equation: **$3x + 4 = 10$**.
You will solve this equation in several ways and discuss the different solution strategies, as follows.

1. Each member of your group should solve the equation $3x + 4 = 10$ using **one** of the five strategies below. Decide as a group and with your teacher who will use which strategy. (If you finish solving using your strategy before everyone else is finished, then solve using another strategy of your choice until everyone in your group is finished with their strategy.) When everyone is finished, go to Problem 2.

   Solution Strategy 1: By Inspection (or Guess and Check)
   Is there a number such that when you multiply it by 3 and then add 4 you get 10? That is, fill in the box: $3 * \theta + 4 = 10$.

   Solution Strategy 2: Symbolic Manipulation
   Subtract 4 from both sides, then divide both sides by 3.

   Solution Strategy 3: Symbolic Manipulation
   Divide both sides by 3. Then continue manipulating until you get $x = \theta$.

   Solution Strategy 4: Table
   Consider $y = 3x + 4$. Enter this function into your calculator. Set up a table with proper step size and range so that you can locate the solution to the equation $3x + 4 = 10$.

   Solution Strategy 5: Graph
   Consider two functions: $y = 3x + 4$ and $y = 10$. Enter these two functions into your calculator. Graph in the proper window so that you can locate the solution to the equation $3x + 4 = 10$.

2. Now work together with your group to explain and analyze the strategies. For each of the strategies, the person who used the strategy should describe or show how it works and explain why it works, so that everyone understands. Then everyone should discuss its advantages and disadvantages. Record your thinking below.

Solution Strategy 1: By Inspection
Is there a number such that when you multiply it by 3 and then add 4 you get 10?
That is, fill in the box: $3 * \theta + 4 = 10$.

How and why it works:

Advantages:

Disadvantages:

Solution Strategy 2: Standard Symbolic Manipulation
Subtract 4 from both sides, then divide both sides by 3.

How and why it works:

Advantages:

Disadvantages:

Solution Strategy 3: Non-Standard Symbolic Manipulation
Divide both sides by 3. Then continue manipulating until you get $x = \theta$.

How and why it works:

Advantages:

Disadvantages:

Solution Strategy 4: Table

Consider y = 3x + 4. Enter this function into your calculator. Set up a table with proper step size and range so that you can locate the solution to the equation 3x + 4 = 10.

How and why it works:

Advantages:

Disadvantages:

Solution Strategy 5: Graph

Consider two functions: y = 3x + 4 and y = 10. Enter these two functions into your calculator. Graph in the proper window so that you can locate the solution to the equation 3x + 4 = 10.

How and why it works:

Advantages:

Disadvantages:

SUMMARIZE

The equation 3x + 4 = 10 is an example of a *linear equation*. Summarize your work on this task by answering the following questions.

a. Describe connections among the five strategies used to solve the equation. Also describe similarities and differences among the strategies.

b. Is there a "best" strategy, one that should always be used on all problems? Explain.

c. For each strategy, think of a particular linear equation, or a problem or situation that involves solving a linear equation, for which that strategy is particularly useful or appealing.

d. Do you prefer some methods over others? Why?

e. Are there other ways to solve a linear equation? Describe any others you know.

Activity 5. Algebra Reasoning Task 1
Reasoning About $(a + b)^2$

Time: 45 minutes

Overview and Rationale

This activity begins with an analysis of the "freshman's dream," $(a + b)^2 = a^2 + b^2$, one of the more common symbol manipulation errors made by high school students. Participants engage in reasoning and proof, and analysis of student work, within the context of symbol manipulation, emphasizing that symbol manipulation can and should be taught meaningfully.

Connection to Other Activities and the Whole Day

This activity illustrates well both the algebraic theme and the process theme for the day by incorporating important symbol manipulation with related reasoning and proof. In addition, the activity foreshadows the main analysis of student work activity that immediately follows.

Conducting the Activity

- **Grouping**
  Groups are formed so that at least one person with secondary mathematics teaching preparation is in each group.
- **Discussion Method**
  A transparency containing the student work, $(a + b)^2 = a^2 + b^2$, followed by the student work analysis questions, is used to introduce the activity. Each group analyzes the work using the analysis questions. After the group analysis, three small groups present to the whole group a summary of their discussion relative to one each of the three analysis questions.
- **"Proof Without Words" Method**
  Demonstrate the NCTM Illuminations applet that shows the geometric justification of $(a + b)^2 = a^2 + 2ab + b^2$. Have each group complete the "Squaring a Binomial" activity sheet in which they provide explanations for each step in the geometric justification provided by the applet.
- **Closure**
  Briefly discuss the reasons for the steps calling on each group to contribute some of their explanations. Relevant figures are provided.
- **Possible Extension**
  If time permits, groups could work on designing and explaining similar geometric justifications of $(a - b)^2 = a^2 - 2ab + b^2$ and/or $(a - b)^2 = a^2 - b^2$. Figures are provided.
- **Time**
  Allow 15 minutes to introduce the activity and group work on $(a + b)^2 = a^2 + b^2$, Allow 10 minutes for reports, 5 minutes to view the applet, and 15 minutes to work on the proof without words, including discussion. Total: 45 minutes.

Materials

- $(a + b)^2 = a^2 + b^2$ overhead
- Applet from Illuminations website: http://illuminations.nctm.org/tools/tool_detail.aspx?id=19
- Activity sheets for "proofs without words"

TM-7 (Overhead)

Analyzing Student Work:

$$(a + b)^2 = a^2 + b^2$$

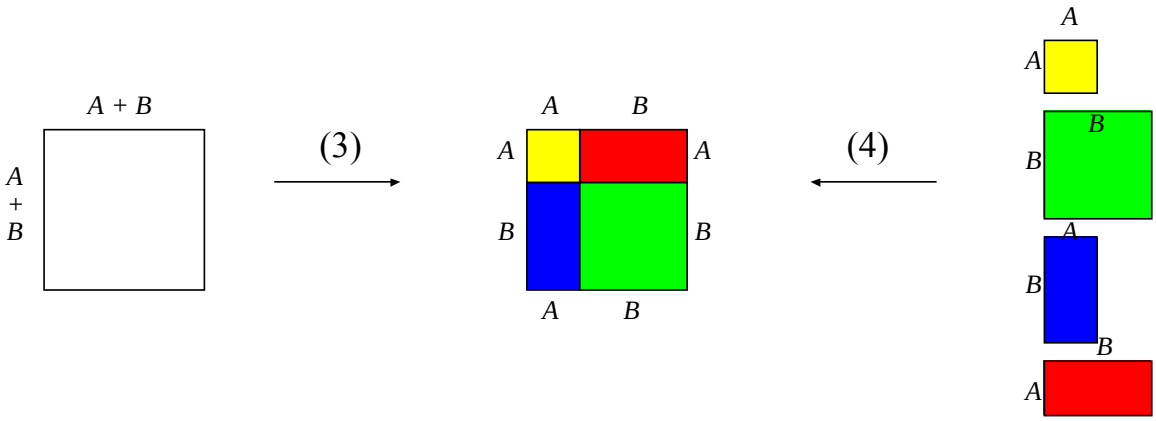
- This work is interesting because it illustrates typical student work on an important algebraic computation.
- Discuss how the student is reasoning and using symbols in this problem. How do you know?
- What questions might you ask to find out more about what the student is thinking?
- If there is a student error, discuss how you might help the student correct the error.

TM 8 (Handout)

Squaring a Binomial

$(A + B)^2$ =?? $A^2 + 2AB + B^2$

(1) (2)



Explain the logical connection indicated by each of the four arrows.

(1)

(2)

(3)

(4)

Choose positive numerical values for *A* and *B* and show that the above symbolic relationship holds true for the numbers you chose.

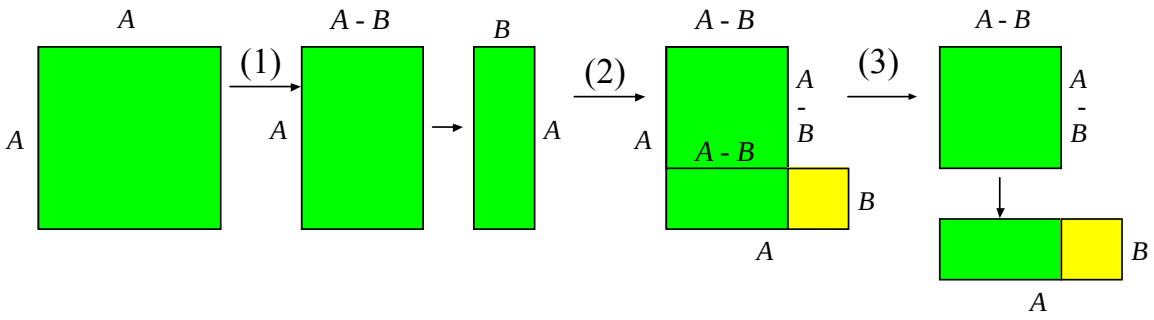
TM 9 (Handout)

Proofs without Words

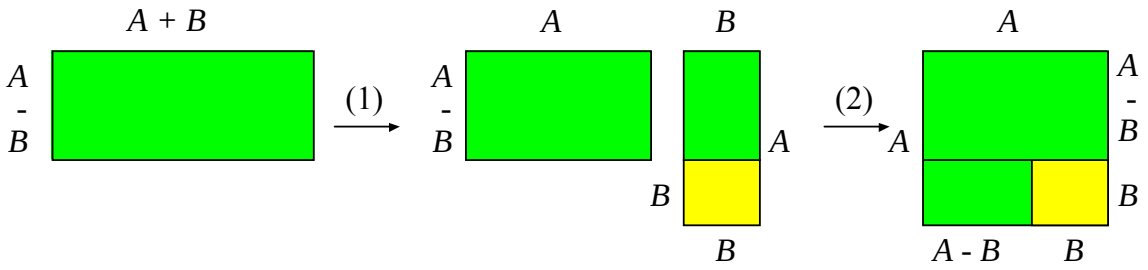
Algebraic Reasoning

Explain the algebraic relationships and reasoning shown below:

$$A^2 \quad - \quad 2AB \quad + \quad B^2 \quad = \quad (A - B)^2$$



$$(A + B)(A - B) \quad = \quad A^2 - B^2$$



Activity 6. Analyzing Student Work

Time: 35 minutes

Overview and Rationale

Part of the participants' assignment was to bring some interesting student work. In this activity, they report what they found to fellow participants and analyze the work from a perspective of assessing and improving student understanding.

Connection to Other Activities and the Whole Day

The first Algebra Reasoning Task leads into this task, as it used the same analysis questions that will be used in this task. This activity is also consistent with the ESC focus on finding out what students do and do not understand and building on their understanding.

Conducting the Activity

- **Grouping**
  Groups are formed so that at least one person with secondary mathematics teaching preparation is in each group.
- **Introduction**
  In this case, one participant had agreed in advance to report on his or her student work as a model for others. He presented his work to the whole class and led a discussion of the work based on the following analysis questions. Discuss how the student is reasoning and using symbols in this problem. How do you know? What questions might you ask to find out more about what the student is thinking? If there is a student error, discuss how you might help the student correct the error.
- **Group Discussion Method**
  Working in small groups, each person presents their student work and explains why it was chosen. The group then discusses the work based on the above analysis questions.
- **Closure**
  If time permits, each group may report to the whole group about their discussion related to one sample of student work. If not, at least highlight that the focus of this activity was on assessing and building upon what students understand about the mathematical topic at hand.
- **Time**
  Allow 10 minutes for the introductory presentation and analysis, 20 minutes for groups to present and analyze participants' student work (assuming 3-4 samples of student work per group), and 5 minutes for closure. Total: 35 minutes

Materials

- Student work that participants provide.

TM-10 (Handout)

Analyzing Student Work: From a Classroom

(From Day 2 Assignment)

- Each person presents student work and briefly describes it. (Why did you choose it? What is the classroom context of the student work?)
- The group analyzes the work together focusing on these points:
  - Discuss how the student is reasoning and using symbols in this problem. How do you know?
  - What questions might you ask to find out more about what the student is thinking?
  - If there is a student error, discuss how you might help the student correct the error.
- Allow about 5 minutes per group member. Then choose one example of student work to present to the whole workshop group.

Activity 7. Algebra Problem-Based Instructional Task 2
Solving Quadratic Equations

Time: 55 minutes

Overview and Rationale

During this day's focus on algebraic symbols, we will investigate two classic problems in high school algebra–solving linear equations and solving quadratic equations. We will see how these formal tasks using algebraic symbols can and should be carried out meaningfully and with a problem-based approach. This solving quadratic equations task emphasizes the connections among solutions, factors, and graphs. In addition, participants see that the form of the quadratic formula can directly give information about the graph of the related quadratic function.

Connection to Other Activities and the Whole Day

This activity complements the linear equation activity. Also, it ties in with the algebra reasoning tasks, both of which involve quadratic expressions. Note that in Day 1 we worked with linear and exponential functions, while in Day 2 we are working with linear and quadratic functions. Thus, we are addressing fundamental function families in beginning algebra during these workshops.

Conducting the Activity

- **Grouping**
  Groups are formed so that at least one person with secondary mathematics teaching preparation is in each group.
- **Introduction**
  Briefly discuss the points above in the "Overview" and "Connection" sections.
- **First Part: Solving Quadratic Equations – Solutions, Factors, Graphs**
  Groups work on the "Solving Quadratic Equations" activity sheet. Make sure they understand that they should complete all missing parts in the table, and they should discuss connections among the columns, that is, among equations, functions, graphs, x-intercepts, and factors. When most groups are finished, discuss in the whole workshop group by having each small group report on their work on one *row* of the table. Then discuss and summarize the connections among columns.
- **Second Part: The Quadratic Formula**
  Groups work on the "Quadratic Formula" activity sheet. The key point here is for participants to realize that the first fraction in the two-fraction form (i.e., -b/2a) immediately shows the x-coordinate of the vertex. After most groups have finished, have one group present their solutions to the whole group, then discuss.
- **Closure**
  Be sure to discuss the following points: (a) what does it mean to solve a quadratic equation, (b) how do you solve a quadratic equation, (c) what are the connections among solving, factoring, and graphing, (d) different algebraic forms give different information, (e) factoring changes an addition problem into a multiplication problem, with accompanying properties (like ab=0 iff a=0 or b=0), and (f) symbol manipulation skill, at an appropriate level, is important, can and should be done meaningfully, and should be based on and extend understanding.
- **Time**
  30 minutes work and 10 minutes discussion for the first part, 10 minutes work and 5 minutes discussion for the second part. Total: 55 minutes.

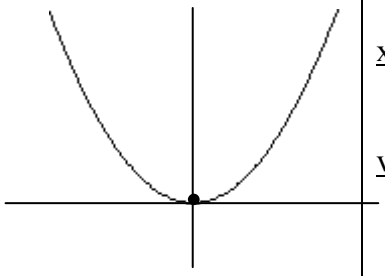
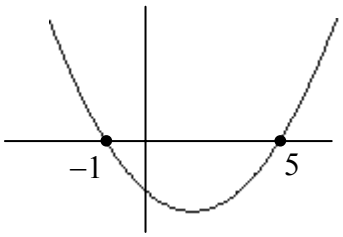
Materials: "Quadratic Equations" and "Quadratic Formula" Handouts, graphing calculators

TM 11 (Handout)

Solving Quadratic Equations

Connections Among Solutions, Factors, and Graphs

Complete this table.
Describe connections among the five columns.

Solve Equation	Related Function	Related Graph	x-intercepts and vertex	Factored Form
Solve: $x^2 = 0$ Solution: $x = 0$	$y = x^2$		x – int: (0, 0) vertex: (0, 0)	$x \bullet x = 0$
Solve: $x^2 - 4 = 0$ Solution: $x^2 = 4$ $x = \pm 2$			x – ints: (–2, 0), (2, 0) vertex:	$(x - 2)(x + 2) = 0$
Solve: $x^2 - 4x = 0$				
Solve: $x^2 - 4x - \square = 0$				
Solve: $x^2 - 5x + 2 = 0$				

TM 12 (Handout)

The Quadratic Formula

Solving the General Quadratic Equation

Examining the Form of the Formula to Obtain Information about the Graph

1. Suppose you want to solve $ax^2 + bx + c = 0$.
The solution of this general quadratic equation, using the quadratic formula, is:

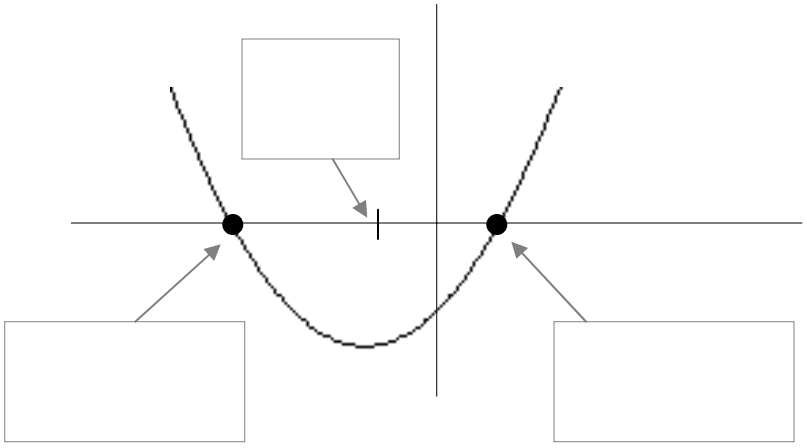
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

which can also be written as:

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}.$$

Explain why these two forms of the quadratic formula are equivalent. (Do not prove the quadratic formula, just explain why the two forms shown above are equivalent.)

2. Below is the graph of $y = ax^2 + bx + c$. Fill in the boxes below, showing the x-intercepts and the vertex.



3. Which of the two forms of the quadratic formula in Problem 1 more clearly shows the location of the vertex of the graph? Explain.

Activity 8. Algebra Reasoning Task 2
Completing the Square

Time: 30 minutes

Overview and Rationale

This activity provides more work with reasoning and geometric representations of algebraic ideas. It also provides some insight into the techniques that underlie the quadratic formula. As in the first algebraic reasoning task, participants engage in reasoning and proof within the context of symbol manipulation, emphasizing that symbol manipulation can and should be taught meaningfully.

Connection to Other Activities and the Whole Day

This activity illustrates well both the algebraic theme and the process theme for the day by incorporating important symbol manipulation with related reasoning and proof. In addition, the activity builds on the previous reasoning task and the solving quadratic equations activity.

Conducting the Activity

- **Grouping**
  Groups are formed so that at least one person with secondary mathematics teaching preparation is in each group.
- **Introduction**
  In whole group, the workshop facilitator introduces symbolically the method of completing the square using the transparency provided. He then demonstrates the completing the square applet on the NCTM Illuminations website.
- **Discussion Method**
  Participants work in small groups using the activity sheet that calls for them to justify each step in the geometric representation of completing the square as done in the applet.
- **Closure**
  Briefly discuss the reasons for the steps calling on each group to contribute some of their explanations. Relevant figures are provided.
- **Possible Extension**
  If time permits, groups could examine and explain the Pythagorean Theorem "proof without words" applet from the Illuminations website.
- **Time**
  Allow 5 minutes to introduce the activity and 15 minutes for group work, Allow 10 minutes for reports. Total: 30 minutes.

Materials

- Completing the square overhead
- Applets from Illuminations website:
  http://illuminations.nctm.org/tools/proofs_without_words3/proofs_without_words3.asp
  http://illuminations.nctm.org/tools/proofs_without_words2/proofs_without_words2.asp
- Activity sheet for the Completing the Square "proof without words"

TM-13 (Overhead)

Completing the Square

Solve: $x^2 = 16$

Solve: $(x-3)^2 = 16$

Solve: $x^2 + 6x = 10$

Complete the square:

- **Take half of 6. Square it. Add the result to both sides of the equation.**

$$x^2 + 6x + 9 = 10 + 9$$

- **The left side will then be a square, so you can take the square root of both sides.**

$$x^2 + 6x + 9 = (x + 3)^2 = 19$$

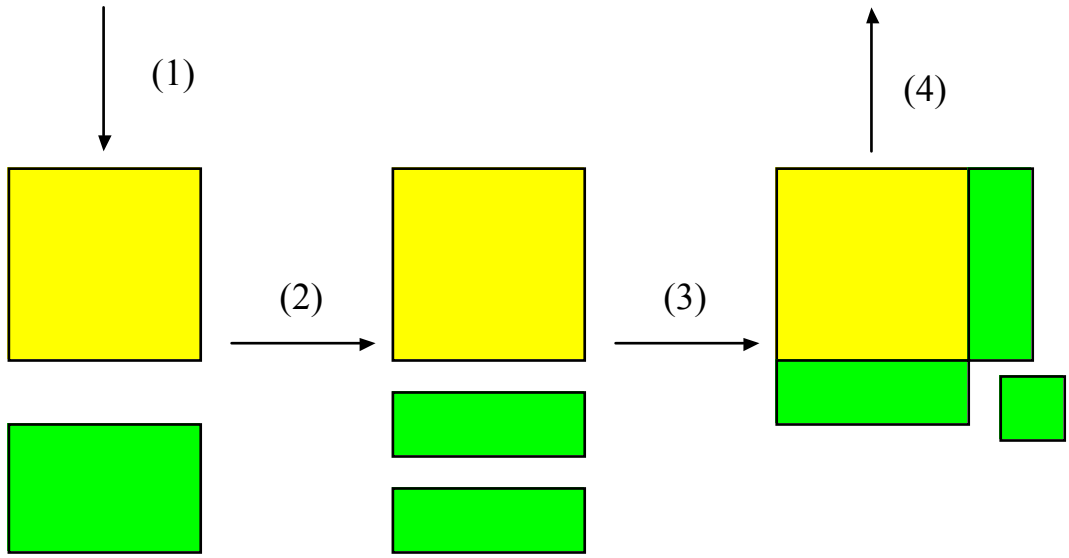
$$x + 3 = \pm\sqrt{19}$$

$$x = -3 \pm \sqrt{19}$$

TM 14 (Handout)

Completing the Square

$X^2 + AX$ =?? $\left(X+\frac{A}{2}\right)^2 - \left(\frac{A}{2}\right)^2$



Explain the logical connection indicated by each of the four arrows.

(1)

(2)

(3)

(4)

Choose a positive numerical value for *A* and show that the above symbolic relationship holds true for the number you chose.

Activity 9. Reflecting on the Day Through the Readings

Time: 20 minutes

Overview and Rationale

This small group discussion is meant to examine the day's activities through the lens of the assigned readings.

Connection to Other Activities and the Whole Day

This activity comes near the end of the day. The intent is to have participants recognize the many examples from the day's activities that relate to the assigned readings.

Conducting the Activity

- **Grouping**
  Groups are the same mix as those in the Discussion of Assigned Readings earlier in the day.
- **Discussion Method**
  Each group examines the list of important points from one of the assigned readings and identifies examples from the day's activities that illustrate these points. Groups then report to the whole group.
- **Time**
  Allow 10 minutes for discussion and 2 minutes for each group report. Total: 20 minutes

Materials

- Lists of important points from readings generated on chart paper in the earlier readings activity

TM 14 (Handout)

Day 2 Readings - Group Discussion Notes from Chart Paper

October 2004

What Research Says About the NCTM Standards" by Hiebert in *A Research Companion to Principles and Standards for School Mathematics*, pages 5 – 23
Research cannot prove what is best
Need to look at multiple perspectives, quality of research, conditions
Students learn what they have opportunity to learn
US curriculum provides few opportunities for students to do challenging problems requiring reasoning, communication and proving
Almost all students can learn more mathematics than they are learning now
Teachers can teach conceptual approach without sacrificing skill proficiency
Finding a balance of traditional procedures and conceptual understanding is key
Instructional programs that emphasize conceptual development, with goal of understanding can facilitate significant learning without sacrificing skill proficiency
Implementation with fidelity is essential
Opportunities for teachers to learn as well as students is necessary
Students learn what they are given the opportunity to learn
Student framework for understanding is fragile—teachers need to reinforce connections/conceptual understanding
Research by itself does not select standards but it can influence process
TIMSS traditional US curriculum repetitive, unfocused, undemanding—dealing with calculating and defining
The "basics" as we've long defined them are limited and shallow
Helps school districts look at different topics that are important to that school district
Focusing more on standard-based processes in teaching. Learn "why" not just "how to"
Standards and research-no clear connection
Just because research cannot prove one way is best does not mean that research is irrelevant
Teachers need to know student outcomes before instruction
Research-make informed decisions
Why do some things work and others do not
Drives decisions
Prevent unwitting mistakes
Standards are driven by research but are not research-based
Students learn what they have on opportunity to learn-teachers also
Research by itself does not select standards. But it can influence the process by questioning assumptions, uncovering deficiencies, revealing possibilities and creating new needs
Chapter 13, "Phasing Problem-Based Teaching into a Traditional Educational Environment" by Copes & Shager in *Teaching Mathematics through Problem Solving Grades 6 – 12*, pages 195 – 205
Chapter 13
Make changes when you are only slightly nervous about them
Teacher as facilitator "student centered"
Choose problems that will carry the mathematics you want to teach
"Solve this problem as many ways as you can"

Solutions is not immediately apparent "let math ideas arise as needed to problem-solve"
Group work—observe, listen, question combined work doesn't lower the grade (parents and students worry about this)
Start small (3-5 min. problem warm-up) build as comfort levels increase
Ask students to solve problems in as many ways as possible
SPOSA (problem solving model)
Group work—start with small groups and smaller increments of time. As the teacher knows more and feels competent with the process, you can lengthen time and make larger groups
Students have certain barriers with group work. Group skill building activities should be included
Make sure students understood the math and other vocabulary that is related to the problem
Role of teacher is now a manager at times
In choosing problems, the most important criteria is to make sure the mathematics matches your objective
Problems chosen must be accessible to all students
Time
First give students a problem that they do not know how to solve, then let the mathematical ideas arise as needed to solve that problem
Move from teacher-centered—student-centered classroom
Discuss it
The more knowledgeable the teacher is with the content, comfort level increases
Articulating math means to be specific about what you want to teach (content, eventually process)
SPOSA
Teacher is manager of investigation
Is the problem carrying the mathematics you want to teach?
Multiple solutions
OBSERVE and LISTEN
Add problem-based teaching to classroom in small steps
Cooperative learning including roles and routines of cooperative groups
.org/potent-website @qualityproblems
www.enc.org
"Reasoning and Proof" for Grades Pre-K – 12 (pp. 122- 126) and for Grades 9 –12 (pp. 342-346) in *Principles and Standards for School Mathematics*
"K-5" students make and investigate conjectures and learn about reasoning thru class discussion of claims that other students make
Not limited to just one content area
Applies "cross curricular" logic
By contradiction CSI
Teachers need a solid mathematical background before leading the group
Reasoning and proof aren't "event." They are natural processes
Teachers need to create a safe climate for discussion—reasoning about problems
The habit of asking "why" is essential for students to develop sound mathematical reasoning
Reasoning/proof allows student to move from concrete to abstract
Consider idea, not student with it
Ask questions—seek justification from students

Make and investigate conjectures
Select and use various types of reasoning and methods of proof
Every large problem is a compilation of many smaller problems and reasoning your way thru the smaller tasks is the “thinking thru” the steps to find a logical conclusion
Reasoning and proof shouldn’t be reserved for special times in curriculum and should be integrated
To help students develop thinking and reasoning skills, teachers need to understand mathematics WELL!
Discussing, questioning and listening should be part of every math classroom activity
Create learning environment, prompt students to make investigations
Get students to explain and justify in oral and written format
Should be included throughout all instruction—it is not a special activity
Create learning environments
Questioning, discussing, listening classroom environment
9-12 develop habits of thinking and reason
Critical think (justify, critique others, analyze)
Natural ongoing discussions
Asking why
Only takes one counter example to prove false
Students should make conjectures and prove them
Ongoing
Daily
Ask “does it make sense?” “Is it logical?”
High expectations
Students are expected to explain and justify their conclusions
Teacher background is important-understand math well
Get in the habit of asking why
Asking the right questions
Discovery can come by asking the right questions
Start very concrete-move to more abstract
Able to support or refute
Students explaining their thinking-ability to express themselves-fosters communication
Types of reasoning algebraic, geometric, proportion, statistical
To clarify
To justify-perception, empirical evidence and short chains of deductive reasoning grounded in previously accepted facts
Relate to prior knowledge-a good foundation
Help students develop productive habits of thinking and reasoning
Help students know why
Allow to grow on past experience
Math is at stake, not the students-it is OK to take a risk
Crucial for the teacher to be an expert in mathematics
Students provided multiple opportunities and engaging context for learning
Teachers must instruct how to have productive conversations
Teacher uses a lot of coaching questions

Sub-section in the Algebra Standard for Grades 9 – 12 entitled "Understand patterns, relations, and functions" in *Principles and Standards for School Mathematics*, pages 297-300
Understand linear, quadratic and exponential functions
Be able to represent as a table, equation or graph
Determining which representation is most appropriate based on a comparison of solution methods
Importance of properties of various functions
Posing "what if" questions to probe conceptual understanding
Recursive aspect of now—next has a more prominent role
Representing situations with multiple algebraic symbols
Process standards greatly incorporated into algebra
Multiple representations is critical
Looking at structure of algebraic thinking—be flexible
Deepen understanding of relations and functions
Expand from linear to other families of function
Use technologies tools to study functions
Modeling of functions
Multiple representations
Connections between the representations
Rates of change
Recognizing the appropriate means of problem solving
Vocabulary—intercepts, zeros, asymptotes
Compare/contrast situations that are modeled by functions
Reflection Question: How does the view of algebra and functions in this reading relate to the activities that you completed on Day 1?
Compare and contrast situations
Multiple representation of problem-look at advantages and disadvantages
Some graphed, while others didn't
Become fluent
We learned properties and classes of functions by exploring and looking at steps
Must connect tools and mathematics to real world phenomena
Deeper "understanding" of change
Modeling
Rate of change varies
Use technology
Recursive/direct forms

Activity 10 & TM 16 (Handout)

Reading Assignment for Day 3

1. The algebraic focus for Day 2 was on the use of symbols.
   - **Read** the sub-section in the Algebra Standard for Grades 9 – 12 entitled "Represent and analyze mathematical situations and structures using algebraic symbols" in *Principles and Standards for School Mathematics*, pages 300-303.
   - This reading describes the main algebraic ideas that were the focus of Day 2.
   - **Reflection Question:** How does the view of the use of symbols in this reading relate to the activities that you completed on Day 2?
2. Problem-based instructional tasks are a central part of algebra classrooms envisioned in the ESC program. Selecting and developing such tasks are discussed from different perspectives in the following readings.
   - Read Chapter 5, "Selecting Quality Tasks for Problem-Based Teaching" by Marcus & Fey in *Teaching Mathematics through Problem Solving Grades 6 – 12*, pages 55 – 67.
   - As the title indicates, this reading provides guidelines for selecting high quality problem-based instructional tasks and gives several examples of such tasks.
   - Reflection Question: According to these authors, as teachers select tasks for a problem-based lesson, unit, or course, what questions should they ask about the tasks under consideration?
   - Read Chapter 6, "Problem Posing as a Tool for Teaching Mathematics" by Goldenberg & Walter in *Teaching Mathematics through Problem Solving Grades 6 – 12*, pages 69 – 84.
   - This chapter provides guidelines for asking questions about traditional mathematical tasks that transform the tasks into problem-based instructional tasks.
   - Reflection Question: Refer to Table 6.1 on pages 75-77. From the second column, choose one question about problem A and one about problem B that seem interesting to you. Write out an answer to each question and think about what, if any, mathematical learning was involved.
3. Problem Solving will be the teaching theme for Day 3.
   - Read "Problem Solving" for Grades Pre-K – 12 (pp. 52-55) and for Grades 9 –12 (pp. 334-341) in *Principles and Standards for School Mathematics*.
   - Be prepared to analyze the Day 3 activities in terms of the four goals of the Problem-Solving Standard.
   - Reflection Question: According to these readings, what is the teacher's role in developing and using problem solving in mathematics classes?

TM 17 (Handout)

Practice Assignment for Day 3

Observe at least one algebra lesson in your

collaborating teacher's classroom. Do the following:

- Take notes so that you can describe the lesson during the Day 3 workshop.
- If the lesson illustrates a problem-based instructional task, as in the Marcus and Fey article (see Readings Assignment), explain how so.
- In any case, pose a problem, as in the Goldenberg and Walter article (see Readings Assignment), that would enrich the lesson.
- In particular, examine Table 6.1 on pp. 75-77, choose at least one of the "Useful Questions" in the left column, and generate a problem derived from the context of the lesson you observed, as in the right column of the table.
- Be prepared to describe and discuss the lesson and your proposed problem during the Day 3 workshop.